

## FINAL EXAM (POONEN) - ANSWER KEY

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### Multiple Choice:

- (1) E
- (2) B
- (3) A
- (4) C
- (5) TRUE
- (6) FALSE
- (7) TRUE
- (8) TRUE
- (9) YES, 2
- (10) NO
- (11) NO
- (12) YES,  $\infty$
- (13) YES, 2
- (14) E
- (15) D
- (16) A

(17)

$$\mathcal{B} = \left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \right\}$$

**Note:** The only difference between this problem and what we've usually been doing is that when you apply the Gram-Schmidt process for the eigenspace corresponding to  $\lambda = 0$ , you have to choose:

$$\mathbf{u}_1 = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

And  $\mathbf{u}_2$  is either one of the other eigenvectors you found (just choose your favorite one!)

(18) (a)

$$A_0 = 4, \quad A_1 = -\frac{8}{\pi}, \quad A_3 = \frac{8}{3\pi}, \quad A_5 = \frac{8}{5\pi}$$

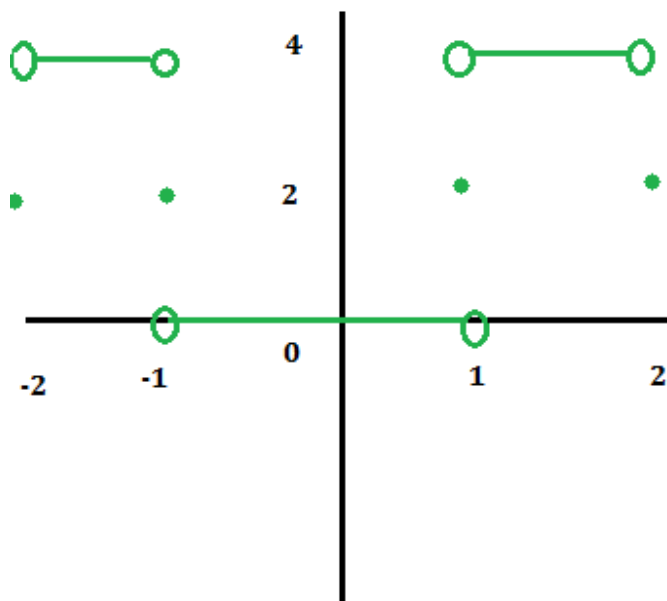
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Date: Monday, December 12th, 2011.

**Note:** All the  $B_m$  terms are 0 because  $f$  is an even function (we want a cosine series)

(b)  $\frac{0+4}{2} = 2$

54/Practice Exams/Poonengraph.png



(19) Use generalized eigenvectors:

$$\mathbf{x}(t) = e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + te^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{3t} \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

(20)

$$u(x, t) = 5e^{-\frac{t^2}{2}1^2} \sin(x) + 7e^{-\frac{t^2}{2}(2)^2} \sin(2x) = 5e^{-\frac{t^2}{2}} \sin(x) + 7e^{-2t^2} \sin(2x)$$